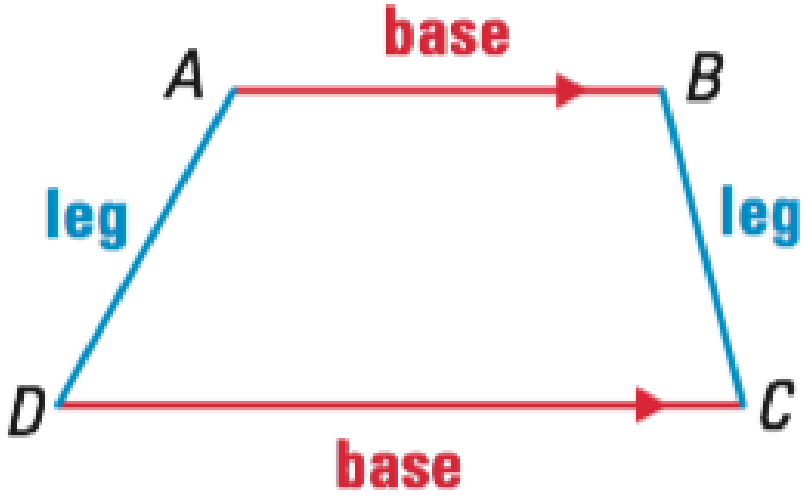
Chapter 6
Quadrilaterals

Section 5
Trapezoids and Kites

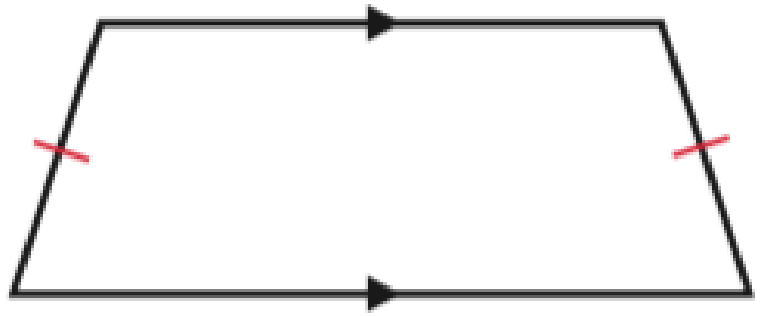
GOAL 1: Using Properties of Trapezoids

A ___trapezoid___ is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the ___bases____.



A trapezoid has two pairs of __base angles_. For instance, in trapezoid ABCD, <D and <C are one pair of base angles. The other pair is <A and <B.

The nonparallel sides are the ___legs___ of the trapezoid.



isosceles trapezoid

If the legs of a trapezoid are congruent, then the trapezoid is an ___isosceles trapezoid__.

THEOREMS

THEOREM 6.14

If a trapezoid is isosceles, then each pair of base angles is congruent.

$\angle A \cong \angle B, \angle C \cong \angle D$

THEOREM 6.15

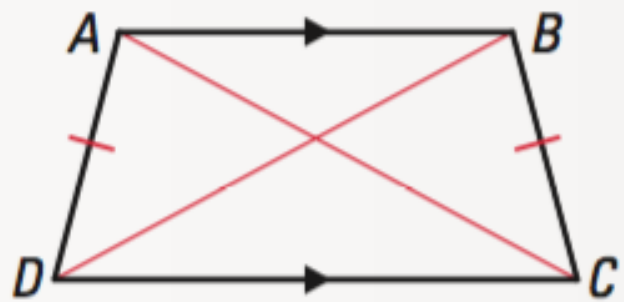
If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

ABCD is an isosceles trapezoid.

THEOREM 6.16

A trapezoid is isosceles if and only if its diagonals are congruent.

ABCD is isosceles if and only if $\overline{AC} \cong \overline{BD}$.



$\overline{AD} \cong \overline{BC}$

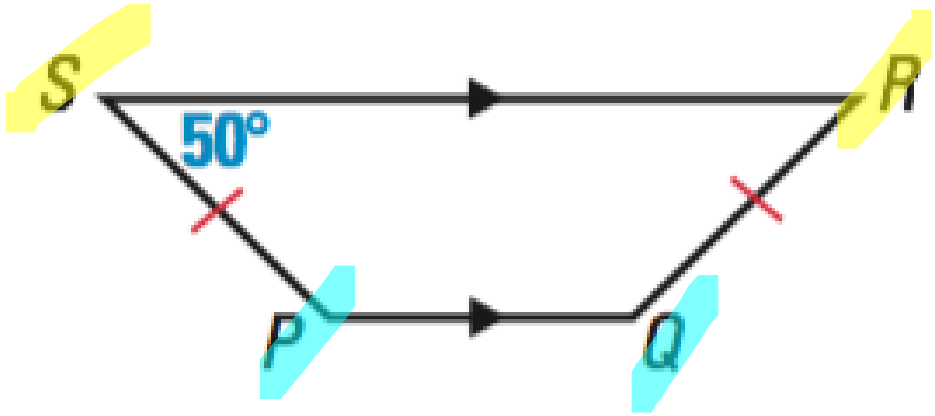
Example 1: Using Properties of Isosceles Trapezoids

PQRS is an isosceles trapezoid. Find m<P, m<Q, and m<R.

m<R = 50*

m<P = 180 – 50 = 130*

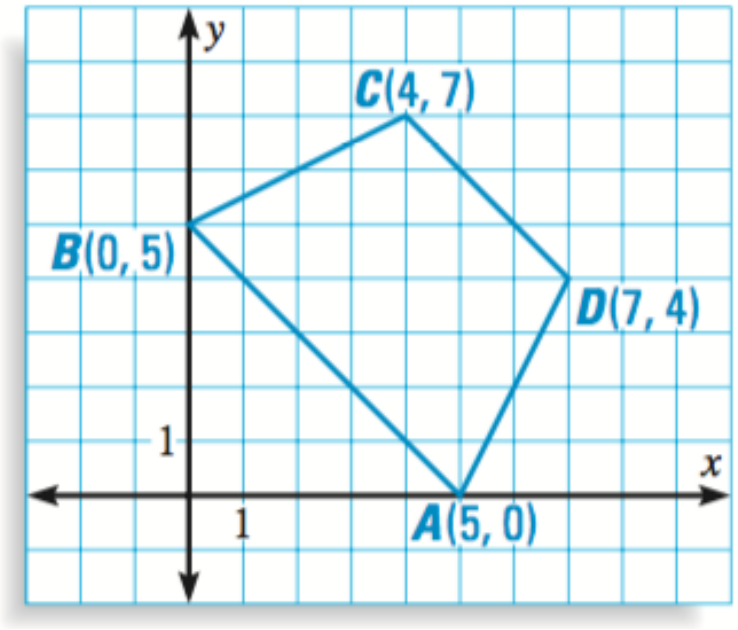
m<Q = 130*



Example 2: Using Properties of Trapezoids

Show that ABCD is a trapezoid.

→ find all 4 slopes → show only 1 pair of sides is parallel



$AB \rightarrow \frac{0-5}{5-0} \rightarrow \frac{-5}{5} \rightarrow -1$

$BC \rightarrow \frac{5-7}{0-4} \rightarrow \frac{-2}{-4} \rightarrow \frac{1}{2}$

$CD \rightarrow \frac{7-4}{4-7} \rightarrow \frac{3}{-3} \rightarrow -1$

$DA \rightarrow \frac{4-0}{7-5} \rightarrow \frac{4}{2} \rightarrow 2$

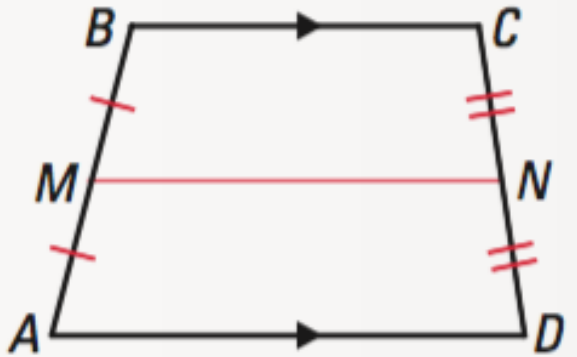
The _____midsegment_____ of a trapezoid is the segment that connects the midpoints of its legs. Theorem 6.17 is similar to the Midsegment Theorem for triangles.

THEOREM

THEOREM 6.17 ***Midsegment Theorem for Trapezoids***

The midsegment of a trapezoid is parallel to each base and its length is one half the sum of the lengths of the bases.

$$\overline{MN} \parallel \overline{AD},\ \overline{MN} \parallel \overline{BC},\ MN = \frac{1}{2}(AD + BC)$$



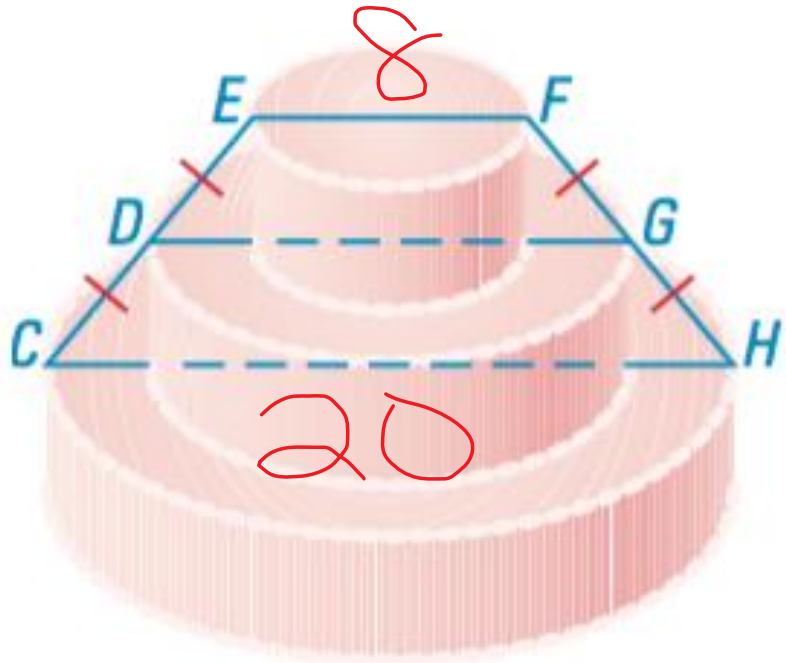
Example 3: Finding Midsegment Lengths of Trapezoids

A baker is making a cake like the one shown. The top layer has a diameter of 8 inches and the bottom layer has a diameter of 20 inches. How big should the middle layer be?

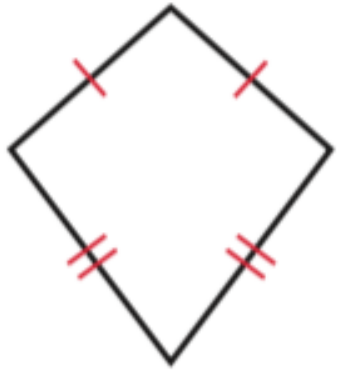
$\frac{1}{2}(8+20)$

$\frac{1}{2}(28)$

14 in.



GOAL 2: Using Properties of Kites

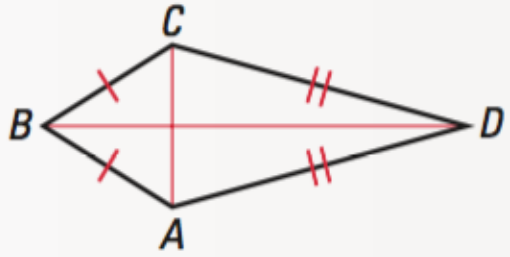


A ___kite___ is a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

THEOREMS ABOUT KITES

THEOREM 6.18

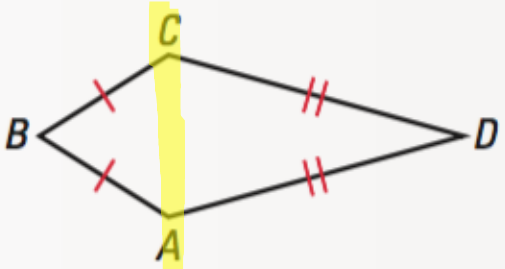
If a quadrilateral is a kite, then its diagonals are perpendicular.



$\overline{AC} \perp \overline{BD}$

THEOREM 6.19

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.



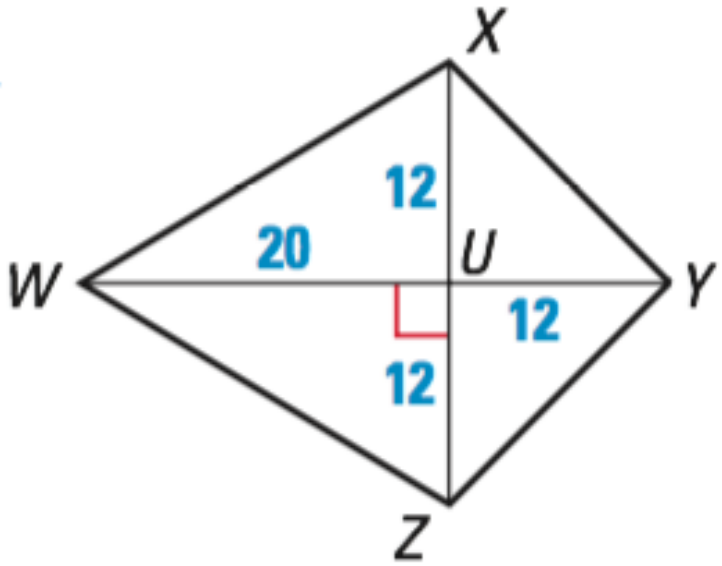
$\angle A \cong \angle C, \angle B \ncong \angle D$

Example 4: Using the Diagonals of a Kite

WXYZ is a kite so the diagonals are perpendicular. You can use the Pythagorean Theorem to find the side lengths.

$$WX = \sqrt{20^2 + 12^2} \approx 23.32$$

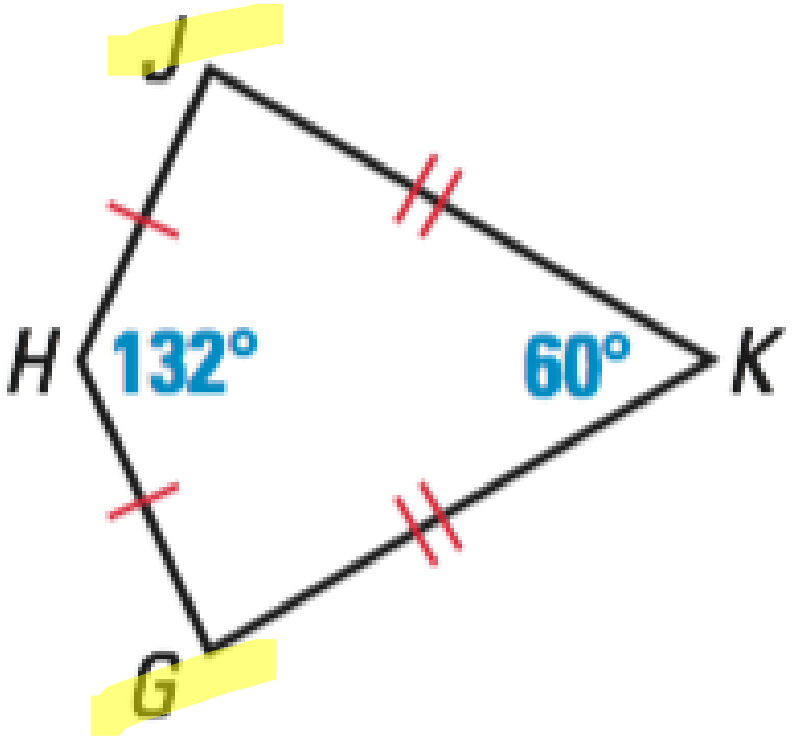
$$XY = \sqrt{12^2 + 12^2} \approx 16.97$$



Because *WXYZ* is a kite, $WZ = WX \approx 23.32$ and $ZY = XY \approx 16.97$.

Example 5: Angles of a Kite

Find m<G and m<J in the diagram at the right.



360 – 132 – 60 = 168

168/2 = 84

m<J = m<G = 84*

EXIT SLIP